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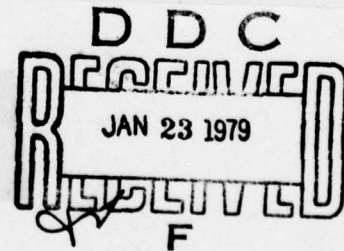# FOREIGN TECHNOLOGY DIVISION

THE HELICOPTER PILOTAGE MODELLING THROUGH THE NUMERICAL METHOD

By

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THE HELICOPTER PILOTAGE MODELLING
THROUGH THE NUMERICAL METHOD

Kazimierz Szumanski

This work deals with the numerical simulation of the helicopter maneuvers which are included in the flight assignment. It also gives the principal foundations, computational model as well as an example of the analysis of the helipopter interrupted take-off.

Besides that the work also includes the suggestion of the employment of the above-mentioned method in other areas which are connected with the evaluation of the dynamics of similar objects being operated by the man.

1. Introduction

The purpose of this study is to find through the computational method a solution of the course of the flight assignment fulfillment which is composed of the helicopter maneuvers' set. Needs for this type of evaluation come from the necessity to deepen the analysis of physical phenomena taking place on the flight different stages. Especially it is justified for helicopters due to their multiparametric steering process which is difficult for a simultaneous evaluation at a given moment. An attempt has been made to create a mathematical projection of the helicopter pilotage technique in the field of the situation evaluation and this of the manner of the maneuver performance.

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The method which has been shown on the example of the evaluation of the helicopter flight dynamics may be also employed for solving the problems connected with the maneuver set of a plane or glider (e. g. a number of acrobatic tricks) as well as for the gliding flight modelling (e. g. the "dolphin" technique) or for the evaluation of other steerable machines belonging to the similar structure, that is the "man operator-machine structure (e. g. car driving). The helicopter motion modelling is becoming justified because of an urgent necessity of projecting and specifying the actual course of phenomena arising during the flight of the helicopter which in general is an object purposely steerable. In this case there is a necessity in the values which are directly being solved in the problem, (e. g. the motion parameters) as well as in the secondary information which can be found during the problem solving, e. g. the helicopter elements' loading as a function of time for an assumed flight profile.

One can list the following fields in which the need for the above-mentioned type of information becomes essential:

1. the resistance analysis (connected both with the ultimate resistance and with the fatigue one) for propulsion (e. g. transmissions; the loading range is necessary) and for the fuselage elements;

2. the reliability analysis - the load distribution as a function of time for the typical, standard, medium and extreme flight profiles;

3. designing the helicopter equipment - looking for the response to the operational process as, for instance in the fuel system (the fuel consumption which depends on the engine power as well as on the equipment loading) as a function of time;

4. testing during the flight becomes essential for optimization of maneuvers, for analyzing the extreme case, for evaluation of the maneuvers which are not typical or dangerous during their evaluation through the test in the flight;

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5. For designing new helicopter constructions - for gaining the better analysis of the helicopter flight characteristics on first stages of the project.

The computing method is based on the digital simulation of the helicopter dynamics. It is justified by the problem complication, nonlinearity and a complicated form of the interdependences, connections between the degrees of freedom of the helicopter in space as well as the necessity maintaining high flexibility of the model because of various character of the phenomena which are being modelled.

A process which is fixed in this way enables to realize the following dynamic tasks:

- it serves for the estimation of the purposeful maneuver (the closed system); here the system with the feedback comes into being. This system is supposed to project three types of maneuvers: a) the extreme maneuver (rough, incorrect); b) the typical maneuver; c) the optimal maneuver;

- it can serve as a simulator (the open system) for the pre-set course of the steering function; it has to respond as a course of changes in the parameter set;

- as the a) problem restriction there is a possibility to project the flight with the autopilot whose effectiveness is limited for the pre-set nominal trajectory;

- it has to embrace also the classical methods of the dynamics evaluation such as uncontrolled whirling motions during the linearization of the motion parameter's deviations, e. g. for the evaluation of the free motion frequency or shape.

In works No. [1], [2], and [3] which are mentioned in the references, the steering by means of devices is based on the principle of autopilot, whose general task is compensating the deviations of the actual parameter set from the pre-set one. The deviation evaluation takes place at the actual moment, and then the set of the nominal parameters must be determined at any moment. In works [2] and [3] it is the nominal landing trajectory and its automatic steering is based on the achievement of proper landing by means of continuous correction of the flight trajectory according to the pattern. The possibility of deviation compensation depends on so-called autopilot effectiveness and the complexity of the pre-set flight parameters depends on the autopilot's perfection. As an example of the pre-set (pattern) parameters one can mention simple parameters, such as: maintaining the course, flight height, descent trajectory, etc. or complex parameters, such as: the turning performance, or automatic landing [2] and [3] where a perfect autopilot is indispensable. If in the computer science we can distinguish such terms as the zero condition and deviation evaluation, then in this work the problem lies rather in the field of looking for the zero conditions. When the autopilot is being employed the problem falls in the category of deviation problems. The difference between the flight simulator and the system which has been proposed in this work lies in the simulator itself, because it is an open system responding to the pre-set steering impulses. Therefore it belongs to the component parts of the proposed model.

The benefits of the employment of the proposed model in comparison with the methods emphasized in the literature on this subject are following: 1) the possibility to solve completely the flight extreme and non-typical forms. This goal in the helicopter problem is especially essential, since fragmentary simplified analyses presently applied are not sufficient; 2) taking into consideration the unification of the model and method and improvement and precision of solution of the typical maneuvers and auxilliary problems of dynamics of the helicopter flight.

2. The Model of the Helicopter Dynamics in the Unsteady Flight

Any flight assignment can be presented as a flight profile consisting of the set of the N maneuvers (the examplary flight profile is on Figure 1). Every nth maneuver can be put apart as a typical flight phase, during which from the t_n moment to the t_{n+1} moment the whole principle is in operation or the right to perform this maneuver is acting. During the acceleration it will be, for instance, maintaining a permanent height and the fuselage gradient (which does not exceed 10-15°); for the turn it will represent maintaining the fixed flight beam and flight height; for the climb it will represent the accomplishment of the pre-set height and climb horizontal speed when the complete power excess is being used. All maneuvers begin at the n point, where all parameters of the helicopter motion are defined. These parameters are identical to the final parameters of the preceding maneuver in the chain of the flight phases which are defined by the flight assignment (Figure 1). In the given flight assignment or during the process of solving the isolated n-maneuver there must be given certain typical parameters of the end of the maneuver at the n+1 point; these parameters are being considered as the goal parameters of the pre-set flight phase. In the phase of giving an impetus it will be the speed arising at the end of giving the acceleration, e. g. V_{x1}=40 per hour, for the climb the typical characteristic will be represented by the height, e. g. z_2=10 m and the horizontal speed V_{x2} is equal to 80 km per hour, etc. The remaining parameters of the helicopter motion will be computed on results from the simulation model of the flight dynamics in the process of solving the chosen n-maneuver.

The n-maneuver solving is based on the definition of changes' course as a function of time of the linear and angular parameters of the helicopter motion (location, speed, acceleration) as well as on the steering process being realized by means of the wobble plate of the main rotor (the total and intermittent pitch) and of the tail propeller (the intermittent pitch).
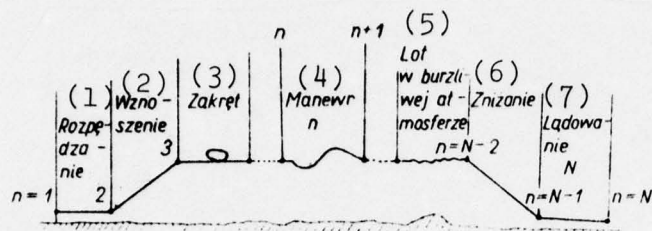
Figure 1. The exemplary flight profile
consisting of the N-maneuver set which
correspond to individual flight phases.
Key: (1) Accelerating; (2) ascending;
(3) turn; (4) manuever n; (5) flight in
turbulen atmosphere; (6) descending;
(7) landing.

The special solving method has been accepted. Its name is the "step by step" solution method where a short period of time - Δt was accepted (e. g. 0.25 second) which secures an additional division of the maneuver on the I amount of periods. The "distance" between the above-mentioned periods of time is Δt.

During the numerical simulation of the chosen maneuver (the simulation concerns the way of solving the flight dynamics as well as the helicopter pilotage simulation) it is prohibited to surpass certain limits, into which because they are common for most maneuvers one can include:

- available engine power;
- design limits imposed on the wobble plate in relation to shaft;
- limitation of the rotor carrying capacity because of jet separation;

Additional limitations arising due to the specifics of the maneuver performed can be following:

- limitations of the helicopter gradient and these of acceleration (the requirement of the flight comfort);

- the limitation of steering rate by means of the steering system (e. g. steering dampers which are used sometimes for the tail propeller or the tempo limitation which is expressed numerically for the evaluation of the "gentle-abrupt" steering way);

The maneuver can be realized in two ways:

1) when the principle of the flight during the maneuver does not change and the end maneuver parameters can be obtained only through continuous correction of the flight parameters. As an example in the case of acceleration the height and gradient are kept stable

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and acceleration ends up at the moment of acquiring the pre-set speed. The same thing happens during the climb to the pre-set height with acceleration lasting to the moment of the established speed being achieved. Continuous correction of the a_z and a_x accelerations' impulses is being carried out in such a way that an available excess power could be used up and this correction makes it possible to obtain the pre-set goal parameters when the remaining parameters change only according to the equations of motion of the helicopter,

2) when the flight principle changes in the isolated n-phase of the flight and there is no possibility of dividing the maneuver into two parts because of their linkage. The moment of the 2nd part's coming out depends on the flight actual parameters according to the principle of the flight realization in the first part. The above-mentioned way can be applied to solving all maneuvers which require, for instance initial acceleration, and later braking the motion, for few parameters which are strictly determined as goal parameters. As an example one can mention the vertical climb over the pre-set height, from hovering at the z_0 height to hovering at z_1 height. The helicopter acceleration takes place before the climb begins being braked. Another example is the autorotation landing. The helicopter flies to a certain, difficult to determine in advance, height by gliding flight, then the process of landing takes place through the kinetic energy of the progressive motion as well as through the inertial energy of the main rotor system in order to reduce the descent. Then during the maneuver's solving besides continuous "directing" towards the goal at the moment of flight realization according to the 1st principle, it is necessary to carry out a continuous computation of "the braking distance" according to the 2nd principle in order to obtain in this case the established goal parameters.

The steering simulation, enabling to select (at the t_1 time) the temporary flight parameters on the grounds of the set-up goal parameters, realizing a maneuver through the direction effect as well as continuous correction of the flight with taking into account the

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actual and expected possibilities and limitations of the helicopter system, is called the motion forseeing system. This system simulates the pilot's actions and has to be determined mathematically in details if it must be used for the purpose of computations. In reality the pilot realizes this system in an intuitive way depending on the kind of training, maintaining in approximation the maneuver principal parameters. In the computing program (the scheme is on Figure 2) the goal and principle of the realization of the n-maneuver are coded in the main block which controls the PR_n. The remaining computations which are common for all maneuvers are coded in proper procedures. These procedures are following:

- computation of the necessary power designated as PN;

- computation of the helicopter's temporary equilibrium in the space RS,

- computation of the N rotor's critical carrying capacity;

- computation of the TET wobble plates' location,

- evaluation of the parameter change from the i-point to the i+1 point in the phase of one KR step,

- control over the achievement of the T goal,

- computing the steering derivatives in order to evaluate the D steering reserves,

- evaluation of the limit time or limit location for the previously specified second type F maneuver (very often it is included in the PR_n program).

The next part of the section contains a detailed discussion of blocks for the computational procedure given in the schematic diagram (Figure 2).
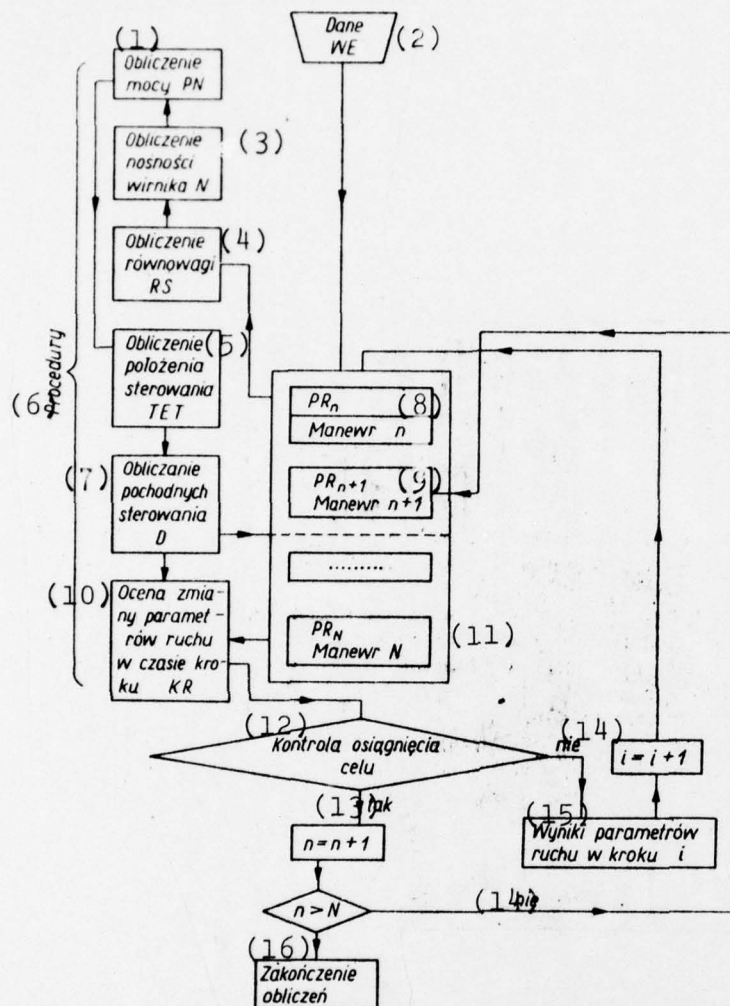
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Figure 2. The scheme of the numerical model organization
for the dynamics simulation of the helicopter flight.
Key: (1) Calculation of power PN; (2) data input; (3)
calculation of lift of rotor N; (4) Calculation of equili-
brium RS; (5) calculation of control position TET; (6) pro-
cedures; (7) calculation of control derivatives D; (8)
maneuver n; (9) maneuver n+1; (10) estimation of change in
parameters of motion during step KR; (11) maneuver N; (12)
monitor accomplishment of task; (13) yes; (14) no; (15)
results of motion parameters in step i; (16) end of calcu-
lation.

2.1. Characterization of individual Blocks of the Maneuver Computational Simulation

2.1.1. The WE (or input) block or the input datum block

It is necessary to submit the estimated principal parameters of the beginning and end of the pre-set typical segment of the maneuver as well as the principal requirements (restrictions) or also how to realize the transition (the motion parameters) between the initial and final point of the set-up segment. More often it will be the symbol referring to the proper procedures.

Therefore, dealing with the input data one should take into consideration the helicopter location in space:

- the x_0, y_0, z_0 co-ordinates of the center of the helicopter mass and the angles (angular displacement) $\Phi, \dot{\Theta}, \Psi$, (roll, pitch, and yaw); component linear speeds V_x, V_y, V_z which were taken from the data concerning the model; limiting values for the rate of movements with the controls, reduced to the wobble plate $\dot{\vartheta}_{0max}, \dot{\vartheta}_{xmax}, \dot{\vartheta}_{ymax}, \dot{\vartheta}_{0tmax}$ (the maximum rate of movement by the total pitch $d\vartheta_0/dt$, of the wobble plate; the wobble plate tilt $d\vartheta_x/dt$, ; the gradient $d\vartheta_y/dt$ and the deflection $d\vartheta_{0t}/dt$). of the total pitch of the tail propeller), Every established rate can be expressed as a certain percentage of limiting rate.

An example of the input data for the n-segment is given in Table 1. The horizontal line in the table of data means that the given parameter is a resultant (passive) one, which provides only the information concerning the course of movement and which is not included into the boundary conditions or limitations. Such resultant (passive) parameters, which from the physical point of view (e. g. the perception of sense and the pilot's working performance) are not able to serve as constraints or as final conditions of the flight
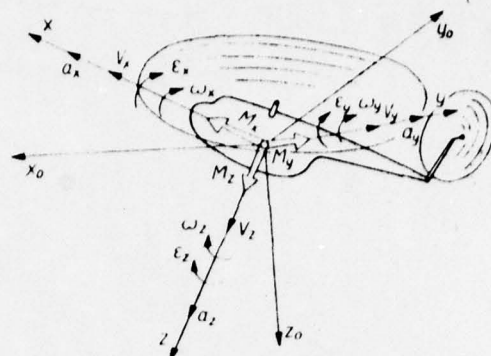
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Figure 3 . The symbol of the
linear and angular velocities
and moments along the axes
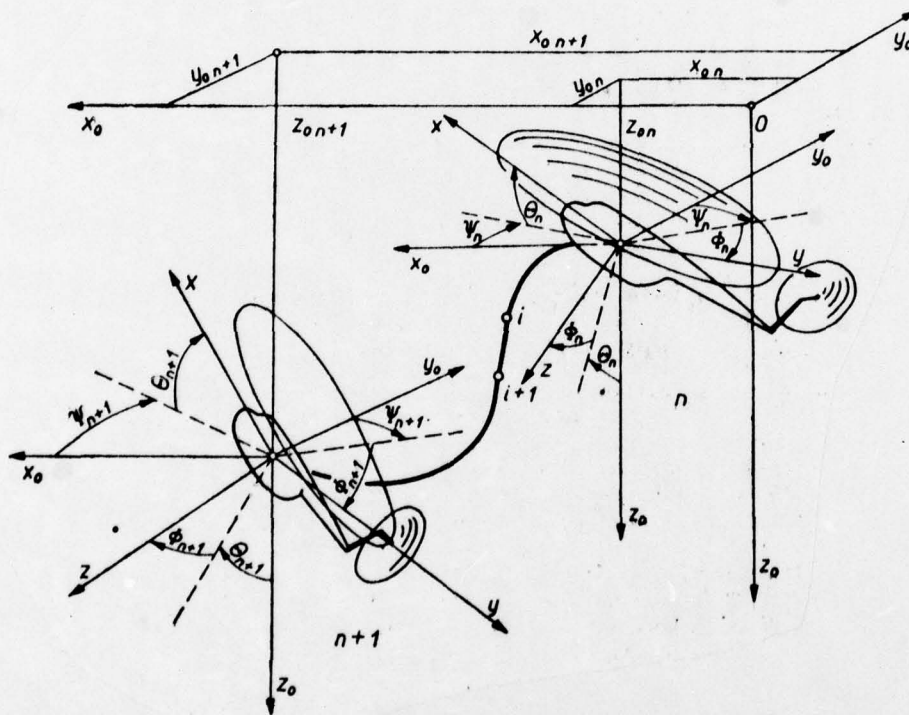connected with the helicopter.

Figure 4.   The scheme of the symbols accepted for
the input data.

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Table 1. The input datum table. For the helicopter exemplary acceleration at the $z_0=3$ m permanent height from 0-30 m/s with the $\theta>-15°$ impassible gradient as well as a further climb on the $z_n=15$ m height at the 80 km/h speed.

(1) Punkty toru lotu	Położenie — współrzędne liniowe (2)			(3) Prędkości (liniowe)		
	x_n	y_n	z_n	V_{xn}	V_{yn}	V_{zn}
	m	m	m	km/h	m/s	m/s
n	0	0	3	0	0	0
(6) Program przejścia od odcinka n	—	—	—	PR_n	—	—
(7) $n+1$	—	0	3	40	0	0
(7) Program przejścia odcinka $n+1$	—	—	—	PR_{n+1}	—	—
$n+2$	—	—	15	80	0	—

Key: (1) Flight Trajectory Points; (2) Location - Coordinate, Linear; (3) Velocities (Linear); (4) Location Coordinate, Angular; (5) Boundary Steering Rate; (6) The program of Passage from the n-segment; (7) The Program of Passage from the n+1 segment.

Położenie — współrzędne kątowe (4)			(5) Graniczne tempo sterowania			
Φ	Θ	Ψ	$\dot{\vartheta}_{0\,max}$	$\dot{\vartheta}_{x\,max}$	$\dot{\varphi}_{y\,max}$	$\dot{\vartheta}_{t\,max}$
°	°	°	°/s	°/s	°/s	°/s
—	> −15°	—	5	5	5	5
—	—	—	—	—	—	—
—	> −15°	—	5	5	5	5
—	—	—	—	—	—	—
	> −15°	—	5	•5	5	5

Continued from page 12

period, are the values a_x, a_y, a_z which belong to the group of operational parameters; ω_x, ω_y, ω_z, ε_x, ε_y, ε_z belong to this group as well because of the impossibility of physically evaluating their magnitude during the flight. However the above-mentioned values can be included in the group of data in the case of piloting by means of automatic pilot, which can also react to the changes in derivative displacements, or in the case of numerical analysis of a maneuver. At times a_z can serve as a boundary parameter in a "saddle pressure" or accelerometer maneuver, which can happen in the case of sharp turn or pull-up.

The given scheme is an approximate outline of the system of the input block and should be treated as a reference tool with flexible application to the tasks dictated by the intended maneuver.

2.1.2. The PN Procedure. Computation of Necessary Power During the Unsteady Helicopter Flight

The acceleration values are taken from the PR_n program. The helicopter elements' loading is taken from the RS procedure; the motion parameters at point i are taken from the *KP procedure. The* PN procedure is intended to calculate the essential power which is being received by the helicopter. Setting up the quasi-stationary system which is based on the principle that the induced velocity changes are considerably faster than the changes in the parameters of helicopter motion, the necessary power can be computed in the following way:

the resultant thrust of the helicopter main rotor:

$$T = \sqrt{T_x^2 + T_y^2 + T_z^2}$$

where T_x, T_y, T_z are components of the thrust in the helicopter system.

The thrust coefficient:

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$$C_T^{\ddagger} = \frac{8T}{\varrho U^2 R \cdot 5{,}73 k b_0}$$

where: ϱ is the air density;

             U is the speed of the end of the rotor blade;

             R is the radius

             k is the number of blades;

             b_0 is the chord of the blade root for $\bar{r}=0$

The induced velocity

$$v = k_1 \sqrt{\sqrt{\frac{V^4}{4} + v_0^4} - \frac{V^2}{2}}$$

where:

$V = \sqrt{V_x^2 + V_y^2 + V_z^2}$ is the flight speed;

$v_0 = \sqrt{\dfrac{T}{2\pi R^2 \varrho \varkappa}}$ is the speed induced in hovering;

$\varkappa$ - coefficient of the boundary losses.

For low flight speeds, for $\dfrac{V}{v_0} < 1,$ in order to allow for the non-uniformity of the induced velocity field, it is necessary to assume that

$$k_1 = \left[1 + 0{,}15\left(1{,}5 - \frac{V}{v_0}\right)\right]$$

and for

$$\frac{V}{v_0} \geqslant 1; \quad k_1 = 1$$

The flow velocity through the main rotor:

$$V_A = V \sin \tau + v \left(\text{the dimensionless flow velocity} \quad \lambda_A = \frac{V_A}{U} \right)$$

where τ is an angle of the inflow on the plane of the main rotor.

Tangential flow around the main rotor.

$V_T = V \cos \tau$ (the dimensionless velocity of the flow $\mu_T = U$).

The ground-proximity influence for slight tilting ϕ of the main rotor in relation to the Earth can be evaluated by means of factor (7).

$$K_T = 1 + \frac{\varrho \, 5{,}73 \, k b_{0,7} \, U^2 (V \varphi + v)}{64 T R^3 (h_w + z)^2 \left[1 + \left(\frac{V}{v} + \varphi \right)^2 \right]}$$

where: $b_{0.7}$ is the chord of the main rotor blade for $\bar{r} = 0.7$

h_w is the height of the main rotor hub over the helicopter central propeller;

z_t is the height of the helicopter central propeller over the ground.

The ventilating power *of the rotor* in the land proximity *to the ground.*

$$P_{wa} = \frac{T V_A}{K_T}$$

The profile power of the main rotor (as a result of this rotor's rotation)

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$$P_{pw1} = \frac{1}{8} \varrho U^3 R k b_0 \, \delta \left(t_4 + \frac{1}{2} \mu_T^2 \, t_2 \right)$$

as a result of the tangential stream

$$P_{w2} = \frac{1}{8} \left(0.425 \mu_T^2 \, t_1 \, \delta_0 + \mu_T \, t_2 \, \delta + 3 \mu_T^3 \, \delta \right) \varrho U^3 R k b_0 \, V$$

where, the blade profile drag coefficient is:

$$\delta = \delta_0 \left[1 + \frac{5.73^2 C_T^{*2}}{t_3^2 + \mu_T^2 (2 t_2 + t_1 \, t_2)} \right]$$

Increase in power due to the compressibility effect ΔP_s

Ecceeding the Ma number at the tip of the blade

$$\Delta M = \frac{U + V}{a^*} - M_{kr}$$

If $\Delta M \leqslant 0$ then $\quad \Delta P_s = 0$

If $\Delta M > 0$ then $\quad \Delta P_s = 0.115 t_4 \, \Delta M^2 \frac{1}{8} \varrho U^3 R \, 5.73 \, k b_0$

$$\bar{r}_2 = 1 - \frac{1}{k} \sqrt{\frac{C_T}{2}} - \text{ end losses}$$

where $\quad C_T = \dfrac{2T}{\varrho U^2 \pi R^2}$

The blade geometric parameters

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$$l_n = \int_{(\bar{r}_1)}^{(\bar{r}_2)} \frac{b}{b_0} \bar{r}^{n-1} \, d\bar{r} \quad - \quad \text{the blade contour parameter}$$

$$k_n = 4 \int_{\bar{r}_1}^{\bar{r}_2} \frac{b}{b_0} \bar{r}^{n-1} \vartheta_{s0} \, d\bar{r} \quad - \quad \text{the blade torsion parameter}$$

Increase in power as a result of stream separation ΔP_0

Having the lower H_L and upper H_U separation limits one can evaluate

$$\Delta P_0 = \frac{H - H_L}{H_U - H_L} P_{w1}$$

Consequently, the total power consumed by the main rotor is:

$$P_w = P_{w0} + P_{w1} + P_{w2} + \Delta P_s + \Delta P_0$$

The power consumed by the tail propeller

The tail propeller is: $\quad T_t = \dfrac{P_w R}{U_t \, l_t}$

where: l_t is the distance between the tail propeller and main rotor shaft.

The velocity of the inflow on the tail propeller is $V_b = V\sin\tau_t$, where τ_t is the inflow angle on the tail propeller plane (the blade tip). The flow velocity through the tail propeller is:

$$\text{1) } V_t = V_b + v_{t1} = \frac{V_b}{2} + \sqrt{\frac{V_b^2}{4} + \frac{T_t}{1{,}81\pi R_t^2 \varrho}}$$

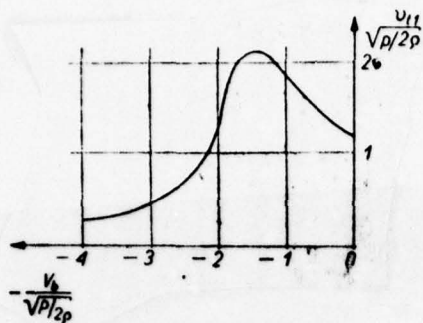For the inflow whose direction is different than the tail propeller induced velocity

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Figure 5. Evaluation of the induced velocity of the tail propeller in the area of the vortex ring.

2) $V_t = -V_b + v_{t1}$ (where: v_{t1} is determined from Figure 5)

where $p = \dfrac{T_t}{\pi R_t^2}$ and consequently

$v_{1t} = \left(\dfrac{v_{1t}}{\sqrt{P/2\varrho}}\right) P/2\varrho$, where R_t is the tail propeller radius.

The tail propeller induced power

$$P_{it} = T_t V_t$$

The tail propeller thrust coefficient:

$$C_{Tt}^* = \frac{8T_t}{\varrho U_t^2 R_t \, 5,53 \, k_t \, b_{0,7t}}$$

where: U_t, k_t, $b_{0.7t}$ = respectively are: the blade tip speed; number of blades, the chord of the tail propeller blade at $0.7\,R_t$.

The profile drag coefficient of the tail propeller blade is:

$$\delta_t = \delta_{0t}\left[1 + \frac{5,73^2 C_{Tt}^{*2}}{t_{3t} + \mu_t(2t_{2t} + t_{1t}\,t_{2t})}\right]$$

where: $\mu_t = \dfrac{V}{U_t}$.

The profile power

$$P_{pt} = \frac{1}{8} U_t^3 R_t k_t b_{0,7t} \delta_t \left(t_{4t} + \frac{1}{2}\mu_t^2\, t_{2t}\right)$$

The power consumed by the tail propeller.

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$$P_t = P_{it} + P_{pt}$$

The total power necessary for the helicopter flight:

$$P = (P_w + P_t)\frac{1}{\xi}$$

where ξ is a coefficient of the power utilization with consideration of losses connected with propulsion of assemblies.

The above-mentioned formulas, although approximate, will be used in an exact model for the evaluation of power increases arising from small changes in motion parameters. Therefore, we should expect precision an order of magnitude greater than with the evaluation of 0 states.

2.1.3. Procedure N. Computing the lift of the main rotor T_{kr} and separation ceilings H_U and H_L

Approximation of the critical lift curve $C_{wU}=f(\mu)$ by analytical dependence:

$$C_{wU} = 0.02 \left[29.8 - \sqrt{26.2^2 - 20\mu_T - 23.6)^2} + (0.5M_0)(0.17 - 0.38\mu_T) + \right.$$
$$+ (\vartheta_{so}\, 57.3 - 7)(0.0005 - 0.00235\mu_T) - 0.008 t_w^*](1 - t_w^*) - 0.005\tau\mu_{Tw}\, 57.3 +$$
$$\left. + (\alpha_{kw} - 14)[(0.018 - 0.024M_0)(1 - 2\mu_T) + 0.004\mu_T] \right]$$
$$C_{wL} = C_{wU} - 0.1 + 0.2\mu_T$$

if $\mu_T > 0.5$ then:

$$C_{wL} = C_{wU} = 5(0.7 - \mu_T)[0.02(29.8 - \sqrt{26.2^2 - 13.6^2})] - (0, -5M_0)\, 0.02 +$$
$$+ (\vartheta_{so}\, 57.3 - 7)(0.0005 + 0.00235\, 0.5) - 0.008 t_w^*[(1 - t_w^*) - 0.005\tau\, 0.5\, 57.3 +$$
$$+ (\alpha_{kw} - 14)\, 0.004\, 0.5]$$

$$\varrho_U = \frac{2T_w}{C_{wU} \, kb_{0,7} \, RU^2} \quad - \quad \text{the air density at the } H_U \text{ separation ceiling}$$

$$\varrho_L = \varrho_U \frac{C_{wU}}{C_{wL}} \quad - \quad \text{the air density at the separation ceiling } H_L$$

Ceilings of the flow separation:

$$H_U = 20\,(0,131 - 0,0004\, t_{oii} - \varrho_U)/(0,131 - 0,0004\, t_{oii} + \varrho_U)$$
$$H_L = 20\,(0,131 - 0,0004\, t_{oii} - \varrho_L)/(0,131 - 0,0004\, t_{oii} + \varrho_L)$$

The critical load carrying ability at altitude H

$$T_{kr} = \frac{1}{2}\,\varrho\, C_{wU}\, kb_{0,7}\, RU^2$$

Symbols:

$M_0 = \dfrac{U}{a^*} \quad -$ the Ma speed of the blade tip while hovering;

$a^* \; -$ sonic speed altitude H;

$\vartheta_{,0} \; -$ geometrical torsion

$\alpha_{kw} \; -$ flow separation angle at the blade tip;

$t_{oii} \; -$ temperature near ground for given type of climate.

2.1.4. The TET Procedure. The location of the control system of the main rotor.

As a result of solving equations of equilibrium, the location of the steering system of the main rotor is calculated.

Assumptions:

- correction of the reverse flow only for the thrust; any blade geometry is included by using the k_n and t_n coefficients; flexibility of blade attachment is included, making it possible to compute unarticulated rotors by introducing equivalent rigidity in a flapping

hinge (according to Figure 6).

- reverse flow for blade hinge and antitorque is not included

- simplified geometry for the induced velocity field is assumed
(according to Glauert);

- it is assumed that flow around a blade element is perpendicular
to the blade axis;

- constancy of the gradient of lift $\frac{dC\dot{z}}{d\alpha}$ is assumed only the first
harmonic of blades motion is considered;

For a given value of thrust T, rotor inclination τ, rolling moment
M_x, pitching moment My, flow V_A, flow V_T, helicopter angular velocities
ω_x and ω_y, which were computed as a result of solving equilibrium
equations RS in a given flight state, the following items are computed:

angle of blade total pitch

$$\vartheta_0 = \frac{C_T^* + k_3 + \frac{1}{2}\mu_0^2 k_1 + (t_2 + 2\mu_0^2)\lambda_0}{t_3 + \frac{1}{2}\mu_0^2 t_1}$$

position of plane of constant angles of incidence

$$a_1 = \frac{\mu_0(2t_3\,\vartheta_0 - 2k_3 - t_2\,\lambda_0) + \frac{2C_{Mx}}{k}}{t_4 - \frac{1}{4}\mu_0^2 t_2} +$$
$$- \left(\bar{r}_2\,\frac{\omega_x}{\omega} + \frac{8\omega_y}{\gamma\omega}\right)\frac{1}{\bar{r}_2^2\left(\bar{r}_2^2 - \frac{1}{2}\mu_0^2\right)}$$

$$b_1 = \frac{\mu_0\,t_3\,a_0 + t_4\,\bar{v}K - \frac{2C_{My}}{k}}{t_4 + \frac{1}{4}\mu_0^2 t_2} - \left(\bar{r}_2\,\frac{4\omega_x}{\omega} - \frac{8\omega_y}{\gamma\omega}\right)\frac{1}{\bar{r}_2^2\left(\bar{r}_2^2 + \frac{1}{2}\mu_0^2\right)}$$

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where $\gamma = 4/\gamma^*$, $\bar{v} = v/U$, $K = 4\mu_T/[3(1{,}2\lambda_T + \mu_T)]$ $\gamma^* = 8I_p/\varrho R^4\,5{,}73\,b_0$, I_p — moment of inertia of blade in relation to flapping hinge.

rotor coning angle:

$$a_0 = \frac{\left(t_4 + \frac{1}{2}\mu_0^2\,t_2\right)\vartheta_0 - \left(k_4 + \frac{1}{2}\mu_0^2\,k_2\right) - t_2\,\lambda_0}{\gamma^* + C_k^*}$$

where $C_k^* = 8k_H/\varrho U^2 R^2\,5{,}73\,b_0$.

ϑ_0 and a_1 are computed by the method of successive approximations; assuming the first approximation for a_1

$$a_1^{(0)} = \mu_T\,\frac{1}{t_4}\,(2C_T^* + t_2\,\lambda_T)$$

we compute $\lambda_0^{(1)}\,\mu_0^{(1)}$ for the assumed $a_1^{(0)}$ and then ϑ_0 and $a_1^{(1)}$ until achieving an accuracy of
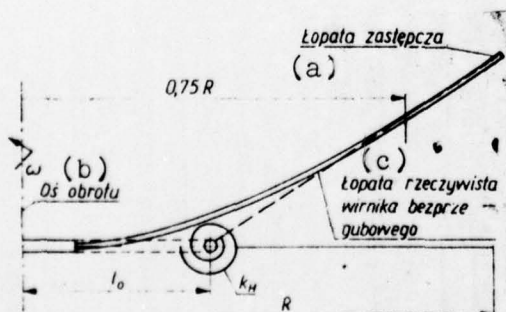
$$|a_1^{(J)} - a^{(J-1)}| \leqslant 0{,}00873$$

Figure 6. Scheme for replacing non-articulated rotor and flexible blades with articulated rotor and rigid blades and with equivalent flexibility k_H in flapping hinge: 1-actual blade of non-articulated rotor; 2 - substitute blade. (a) substitute blade; (b) rotation axis; (c) actual blade of non-articulated rotor.

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which usually happens in the 2nd or 3rd approximation.

the flow components through the rotor on the plane of constant angles
of incidence:

$$V_a = V_A \cos a_1 + V_T \sin a_1$$
$$V_t = -V_A \sin a_1 + V_T \cos a_1$$

dimensionless values

$$\lambda_0 = \frac{V_a}{\omega R}$$

$$\mu_0 = \frac{V_t}{\omega R}$$

wobble plate location in relation to the shaft

$$\vartheta_{ty} = [(a_{1y} - \Theta + \gamma_y) D_1 - (\Phi - \gamma_x - b_{1y}) D_2]/(D_1^2 + D_2^2)$$
$$\vartheta_{tx} = (b_{1y} - \Phi + \gamma_x + D_2 \vartheta_{ty})/D_1$$

where:

$$D_1 = \frac{\bar{D}_1 + \bar{k}\bar{D}_2}{1 + \bar{k}^2} \; ; \quad D_2 = \frac{\bar{D}_2 - \bar{k}\bar{D}_1}{1 + \bar{k}^2}$$

$\bar{D}_1$, $\bar{D}_2$ - the control system coefficients (data from control system
design);

k - the kinematic connection of oscillations and turns.

Location of the wobble plate in relation to the plane of the main rotor
blade tips.

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$$a_{1y} = \frac{a_1 + \bar{k} b_1}{1 + \bar{k}^2}$$

$$b_{1y} = \frac{b_1 - \bar{k} a_1}{1 + \bar{k}^2}$$

inclination and bending of the thrust vector of the main rotor.

$$\gamma_x = \text{arc tg} \frac{T_x}{T_s}$$

$$\gamma_y = \text{arc tg} \frac{T_y}{T_s}$$

2.1.5.   The RS Procedure.   The Helicopter
Equilibrium Computation

The most important problem as well as initial action is to bring
the motion equations into agreement with the physical equivalent of
the phenomenon which is being modeled. The equation system has to
consider both the evaluated motion parameters and the coefficients
which connect them depending on the helicopter design parameters as
well as on its system characteristics. The motion equations' system
in space has to consider the sum of forces and moments in relation
to three axes and it should also consider the equation of motion (or
energy) of internal degrees of freedom, primarily of the system of the
main rotor. In most cases in modern helicopters the additional motion
equation has a lower value, since (contrary to airplanes) in case of
employment of amplifiers in the rotor-to-controls direction the system
is self-braking, which makes superfluous the evaluation of the dynamics
of motion with a "release rudder". To evaluate the above-mentioned
dynamics an addition equation of the control system was necessary,
in the general case the motion equations must include:

1) the motion equation of a helicopter as a rigid body in a system
with an axis connected with the helicopter (the Eulerian equation),

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2) equations of motions of control systems;

3) the motion equation (energy, power balance) of the inertial system of the main rotor;

4) possible equations of deformation of the helicopter

The equation of the motion of the helicopter element δ_m

$$\delta\vec{F} = \delta_m \frac{d\vec{v}}{dt}$$

Completing the integration we will obtain the force equation:

$$\vec{F} = m \frac{d\vec{v}}{dt}$$

and from here we will obtain 3 scalar force equations

$$\delta\vec{h} = \vec{r}\ \vec{v}\delta_m$$

The elementary moment of motion

$$\vec{G} = \frac{d\vec{h}}{dt}$$

and from here 3 scalar moment equations.

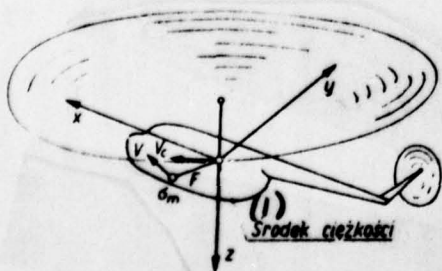For the co-ordinate system according to Figure 7 one can obtain 6 motion equations.

Figure 7. The accepted co-ordinate system. Key: (1) Center of gravity.

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Beginning with a system of axes connected with the helicopter at the center of gravity, we will obtain the following equation system:

$$m(a_x + V_z \omega_y - V_y \omega_z) + mg \sin \Theta = X$$

$$m(a_y + V_x \omega_z - \omega_x V_z) - mg \cos \Theta \sin \Phi = Y$$

$$m(a_z + V_y \omega_x - V_x \omega_y) - mg \cos \Theta \cos \Phi = Z$$

$$I_x \varepsilon_x + (I_z - I_y) \omega_y \omega_z - I_{xy}(\varepsilon_y - \omega_y \omega_z) + I_{yz}(\omega_z^2 - \omega_y^2) - I_{xz}(\varepsilon_z + \omega_x \omega_y) = L$$

$$I_y \varepsilon_y + (I_x - I_z) \omega_x \omega_z - I_{yz}(\varepsilon_z - \omega_x \omega_y) + I_{xz}(\omega_x^2 - \omega_z^2) - I_{xy}(\varepsilon_x + \omega_y \omega_x) = M$$

$$I_z \varepsilon_z + (I_y - I_z) \omega_x \omega_y - I_{xz}(\varepsilon_x - \omega_y \omega_z) + I_{xy}(\omega_y^2 - \omega_x^2) - I_{yz}(\varepsilon_y + \omega_x \omega_z) = N$$

The aero-dynamic forces and moments located on the right side of the equations are non-linear functions of the flight speed, wobble plate deflection, main rotor total pitch, tail rotor pitch and gust speed.

The angular velocities in relation to the axes connected with the helicopter and designated in equations as ω_x, ω_y, ω_z are also non-linear functions of the Euler angles Φ, Θ, Ψ

$$\omega_x = \dot{\Phi} - \dot{\Psi} \sin \Theta$$

$$\omega_y = \dot{\Theta} \cos \Phi - \dot{\Psi} \cos \Theta \sin \Phi$$

$$\omega_z = \dot{\Psi} \cos \Theta \cos \Phi - \dot{\Theta} \sin \Phi$$

The rotor equation of moments (power)

$$P_p + P_i + P_{sz} + P_c = ma_x V_x + GV_z + I\omega\dot{\omega} + P_r$$

where: P_p and p_i are the profile and induced power of the main rotor;

P_{sz} is power lost to parasitic drag;

P_c is power consumed for climb or descent;

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V_z is vertical speed (negative for descent);

I is inertial moment of the inertial system of the main rotor reduced to the rotor axis and revolutions;

ω is the rotor angular velocity;

V_x is flight speed (horizontal);

a_x is the helicopter's acceleration;

P_r is power supplied from propulsion.

The general solution of the equation system can be achieved by the numerical method. It is applicable in cases when the decisive elements of motion remain unknown and simplifying assumptions cannot be applied. Quite often it is necessary to analyze significant motion disturbances (e.g. when evaluating maneuvers), where non-linear dependences and coupling of equations cannot be forseen. The numerical solution process through the operations performed on numbers instead of formula conversion) of the helicopter transient equilibrium is an approximate process.

The "step by step" method is applied (see the KR procedure). Starting from the V_x, V_y, V_z, ω_x, ω_y, ω_z initial values of the velocity as well as the rudders' locations and the rotor thrust components it is necessary to compute the forces, X, Y, Z, M_y, M_z, and after that the values of acceleration a_x, a_y, a_z, ϵ_x, ϵ_y, ϵ_z. Then assuming that the above-mentioned accelerations **during time period** Δt are constant, the new values of velocity obtained by the end of time period Δt are computed. Accepting the new speed values as the output ones for computations during the next time period Δt one can compute in the step-by-step way the entire pattern of motion while considering the
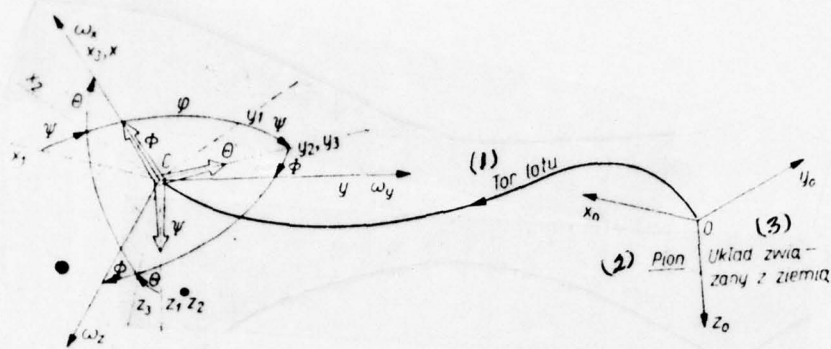
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Figure 8. The helicopter orientation in space. Key:
(1) flight trajectory; (2) Perpendicular; (3) System
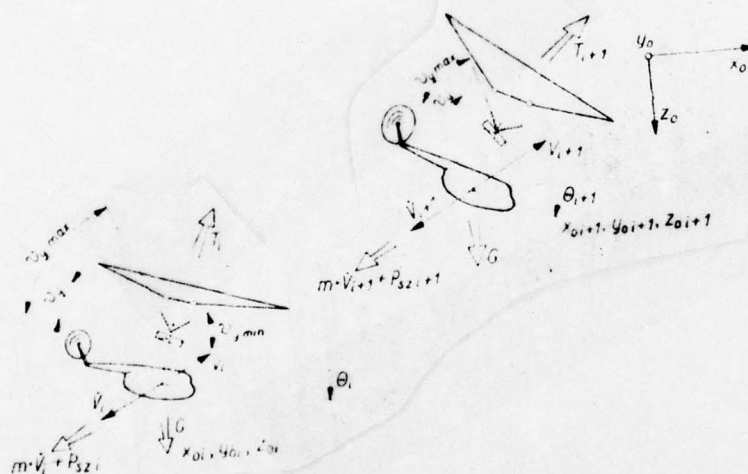connected with the ground.

Figure 9. The motion parameters computed in the KR phase.

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influence of change of forces and without limiting oneself to the case of the linear dependences of forces on the steady motion disturbances (which is very important with significant flight deviations or in flight conditions with flow separation or within the compressibility influence). Changing time period Δt, one can achieve more precise computation results.

2.1.6. The KR Procedure. (The Solution Step for the Time Interval Δt Between t_1 and t_{i+1}

The segment of the flight trajectory with a uniform transition technique points n-n+1 should be divided into a series of time intervals for precisely determining the pattern of variation in parameters of motion between points n and n+1.

This procedure deals with the problem of these parameters' behavior within the **n**-n+1 segment in order to control the impulse of the thrust components ΔT_x, ΔT_y, ΔT_z, ΔT_t of the rotor which are brought into being through the displacement of the control system by $\Delta\vartheta_x, \Delta\vartheta_y, \Delta\vartheta_0, \Delta\vartheta_t$. The accelerations of the helicopter, which is assumed to be constant in the time Δt, correspond to increments in thrust. These accelerations are equal to the accelerations at point i or, in the case of the iterative repetition of computations within an interval of one step, to the mean value computed with the following formula:

$$(a_{xi}^{(j)}+a_{xi+1}^{(j)})/2$$

where j is a succeeding step iteration

At the beginning and end of the i-step the following parameters are analyzed: the location, speed, and acceleration of the helicopter; the values of the aerodynamic and mass load of the helicopter

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components as well as the power consumption through the rotor system. The angular location (Φ, Θ, Ψ) is necessary in order to analyze the wobble plate location in relation to the rotor shaft for the purpose of the determining steering reserve in the following step and the possibility of bringing about a proper impulse. The wobble plate possesses a small range of angular deflections (in relation to the shaft), and consequently the helicopter location in space influences significantly the control over the value and inclination of the thrust vector.

For illustration certain parameters of the helicopter equilibrium computed in the KR phase are shown (See Figure 9).

For small Δt and the average values of the accelerations a_x, a_y, x_2, ε_x, ε_y, ε_z, the parameters of the end of step i+1 can be expressed through the parameters of the beginning of the step:

$$x_{i+1} = x_i + V_{xi} \Delta t + a_x \frac{\Delta t^2}{2}$$

$$y_{i+1} = y_i + V_{yi} \Delta t + a_y \frac{\Delta t^2}{2}$$

$$z_{i+1} = z_i + V_{zi} \Delta t + a_2 \frac{\Delta t^2}{2} \qquad \text{displacements at point i+1}$$

$$\Phi_{i+1} = \Phi_i + \omega_{xi} \Delta t + \varepsilon_x \frac{\Delta t^2}{2}$$

$$\Theta_{i+1} = \Theta_i + \omega_{yi} \Delta t + \varepsilon_y \frac{\Delta t^2}{2}$$

$$\Psi_{i+1} = \Psi_i + \omega_{zi} \Delta t + \varepsilon_z \frac{\Delta t^2}{2}$$

$$V_{xi+1} = V_{xi} + a_x \Delta t$$

$$V_{yi+1} = V_{yi} + a_y \Delta t$$

$$V_{zi+1} = V_{zi} + a_z \Delta t \qquad \text{velocities at point i+1}$$

$$\omega_{xi+1} = \omega_{xi} + \varepsilon_x \Delta t$$

$$\omega_{yi+1} = \omega_{yi} + \varepsilon_y \Delta t$$

$$\omega_{zi+1} = \omega_{zi} + \varepsilon_z \Delta t$$

2.1.7. The D Procedure (derivatives, control gradients)

$$T_{x,y,z} = f(\vartheta_0, \vartheta_x, \vartheta_y); \quad T_t = f(\vartheta_{0t})$$

As a result of the solution of the helicopter equilibrium equations by means of the RS procedure as well as by means of the changes in the motion parameters evaluated in the KR procedure, the location and loading of the main rotor and the flow around it are determined.

T_x, T_y, T_z - load;

V_A, V_T, V_B - flow;

ω_x, ω_y, ω_z - the angular velocities of helicopter roll, pitch, and yaw in relation to the center.

There are no methods yet for the precise computation of the dependences T_x. T_y, $T_z = f(\vartheta_y, \vartheta_x, \vartheta_0)$ for the given parameters of flow around and motion of helicopter in space, which could serve for the determination of the derivatives $\frac{dT_z}{d\vartheta_0}$, $\frac{dT_x}{d\vartheta_y}$, etc. in order to evaluate the thrust impulse caused by the wobble place motion with values $\Delta\vartheta_0, \Delta\vartheta_y$

However, for the given values of load, flow and angular velocity of the rotor in space it is possible to find relatively precisely the increments $\Delta\vartheta_0, \Delta\vartheta_x, \Delta\vartheta_y$ for the given $\Delta T_x, \Delta T_y, \Delta T_z$, or to solve an inverse problem. It allows the following procedure: the necessary power and the positions of the control systems at a given level of the rotor load are computed. Then, for a given increment, e.g. $\Delta T_z = 100$ kG (or acceleration impulse, e.g. Δa_z 1 m/s^2) new powers and positions of the control systems are computed and from this we obtain such required derivatives as:

$$d_P = \frac{\Delta P_n}{\Delta \vartheta_i} \quad \text{where} \quad \Delta P_n = P_{n(T_z + \Delta T_z)} - P_{n(T)}$$

$$d_T = \frac{\Delta T_z}{\Delta \vartheta_i}$$ where, for instance $\Delta T_z = 100 \text{ kG}$ is assumed.

These derivatives will be used both in the evaluation of the margins of control and in the selection of the permissible control impulse in the i-step.

2.1.8. The PR_n Program. Flight on n-flight Trajectory

This program represents a mathematical model for the method of accomplishing of a segmental task. It is constructed to correspond physically to an actual pilot-helicopter system and piloting techniques used. In the program of flight for segment n there must be a coded principle of this flight as well as the goal realization method, the so-called final parameters of the segment. Formulating then, according to the physical sense and realization technique of a proper maneuver, the mathematical model of its realization, attention should be drawn to the following problems:

1) energy distribution (allocation of the available power to climbing, horizontal acceleration and, if necessary, to acceleration of the rotor, etc.) which in reality is accomplished intuitively by the pilot, who sets the values of the parts arbitrarily;

2) connecting the maneuver segments (so-called transitional states) between two different sequential maneuvers;

3) forseeing the helicopter motion, that is, the computation and extrapolation parameters of the current moment in order to evaluate the probability of the reaching goal and to correct the actual control impulses in such a way that the given parameters of the end of the segment could be obtained with a permissible tolerance;

4) the tolerance for achieving the final parameters of motion;

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5)   selection of the helicopter control impulse during step i.

The co-ordinate cuboid (Figure 10), at whose diagonal ends the boundary conditions of helicopter motion are determined, is a unified element of the flight profile which makes it possible to realize virtually any task, which can be divided into segments characterized by similar pilotage techniques. There is always a certain option in switching from point n to point n+1 but in the algorithmized procedure there must be an unequivocally coded method of reaching the required motion parameters, taking as a basis the parameters given at the n-point. This option is limited by the boundary conditions, the assumed rate of control, and the accepted principle of the flight for segment n. If it is difficult to achieve the interconnected parameters of terminal point n+1 in one maneuver computation, the computation should be repeated and the solution obtained by the iterative process. Cases of this kind occur very often when the flight task is being optimized.
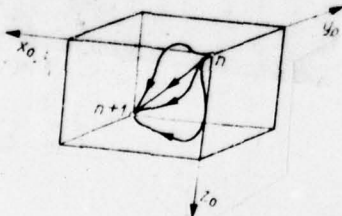

Figure 10. Option flight paths from point n to point n+1.

2.1.9. The F System (prediction). The Model of the Pilot's Action

The system is to evaluate the motion parameters in the immediate future and to provide information on whether and in what way to change the helicopter control functions, based on the anticipated values of the motion parameters, in order to obtain the given characteristic values of the function of the goal (the parameters at point n+1). It is a physical equivalent of the pilot's earliest reaction according to the anticipated motion in order to obtain a definite flight condition. This system, by extrapolating the current parameters of motion, evaluates the effect of the control

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system action. Basing itself on the evaluation of the situation in the step which is under linear consideration or on $_\wedge$approximative *an* function, this system predicts the further pattern of motion. Thus, by correcting the control$_\wedge$every step and advancing correspondingly, *in* it is possible to reach the specific motion parameters at the moment when the maneuver n is over.

In the controlled objects, depending on efficiency of control and the actual parameters of motion one can separate the limiting time interval where there is a possible required change in parameters of the motion when the limiting possibilities of control are used. This value is of a significant importance in goal-oriented control when the pilot's reaction is based on an anticipated flight state and not on a current set of motion parameters.

2.2. Theoretical Bases for Determining limiting time Δt_{gr}

In the general case it is the time necessary for a change in the vector of velocity with value $\Delta \vec{V}$ (Figure 11) where $\Delta \vec{V}$ is the difference between the velocity of the goal target at point $n-\vec{V}_n$ and the velocity at point $i-\vec{V}_i$ with the **assumed** resolution of the maximum acceleration in segment i-n.

$$t_{gr} = \int_{t_i}^{t_n} \frac{v(\Delta)}{1} \, d\Delta$$

The resultant motion of the helicopter in a maneuver is a super-positioning of displacements in relation to its axis. For every degree of freedom one can anticipate the helicopter shifts in the computational program and calculate the limiting time.

For example, the physical picture of changes in the boundary zone which determines the prediction time necessary for further prediction of motion is given in the example of the change in the vector of velocity at point n at 90° during accelerated flight from point n-1 to point n, starting from velocity V = 0 (Figure 13).

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Figure 11. Change in the
vector of velocity $\Delta\vec{V}$ between
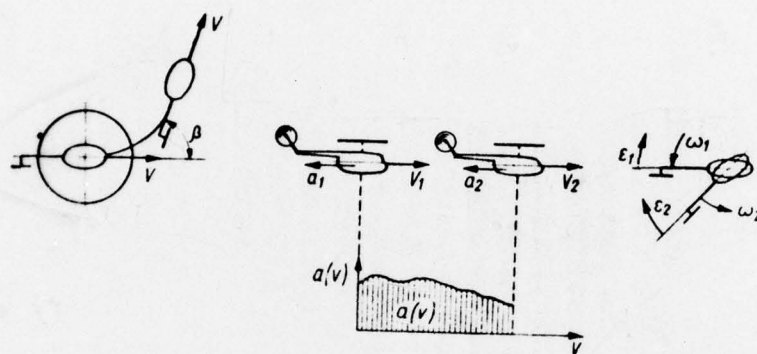velocity $\vec{V}_n$ and the velocity
at a current instant.

Figure 12. Elementary maneuvers as a basis for
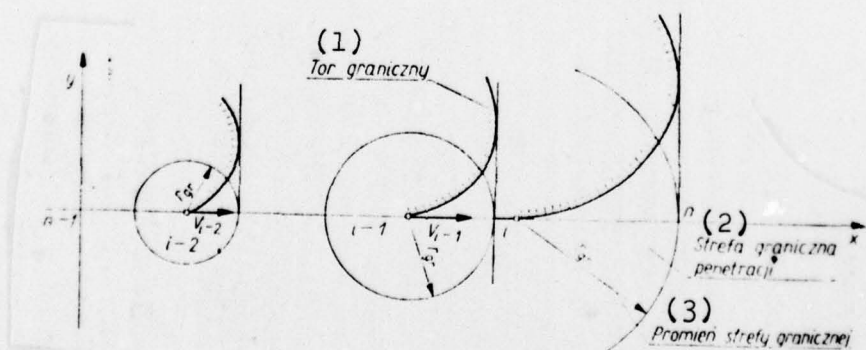determining limiting time Δt_{gr}.

Figure 13. Change in the region and limiting path of the
change in velocity with a change in flight velocity along
the path: 1 - limiting path; 2 - penetration boundary
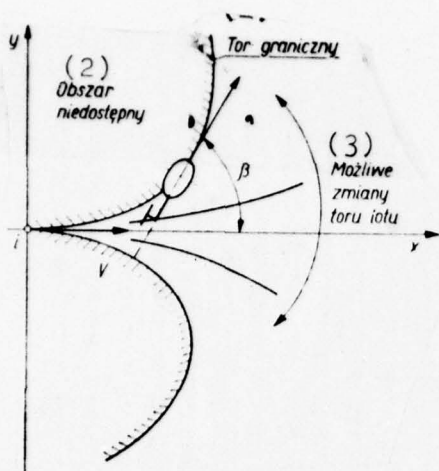zone; (3) boundary zone radius.

Figure 14.   Figure 15.

Figure 14.   Evaluation of the range of permissible change in the
flight path.   Key: (1) inaccessible;region; (2) limiting path;
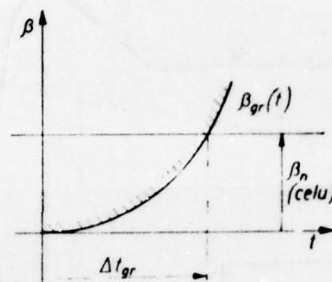(3) of goal.

Figure 15.   Determining Δt_{gr} as a transposition of the solution
of the flight path from Figure 14.

The necessary limiting time can be computed with the following il-
lustration of a change in the direction of flight (Figures 14, 15).
There is a similar representation of the value of acceleration or
braking of linear or angular movements (Figure 16).   The scheme of
action of system F can be illustrated in the example of a helicopter
vertical climb to the preset altitude $Z_n = H$ (Figure 17).   In this case
$x_1 = y_1 = z_1 = x_2 = y_2 = 0$.   Continuous computation of the possibility of braking
the motion makes it possible at the proper time (point A) to decide
about braking the motion and at the point B to decide about the reaching
the given altitude of climb, which in this simple maneuver was the goal
and the boundary condition of the end of segment n+1.

When setting the F system procedure one should consider the fol-
lowing elements: 1 - evaluation of the difference of the current and
goal parameters and the instantaneous limiting margins of control;
2 - the principle of achievement of the goal function (the condition
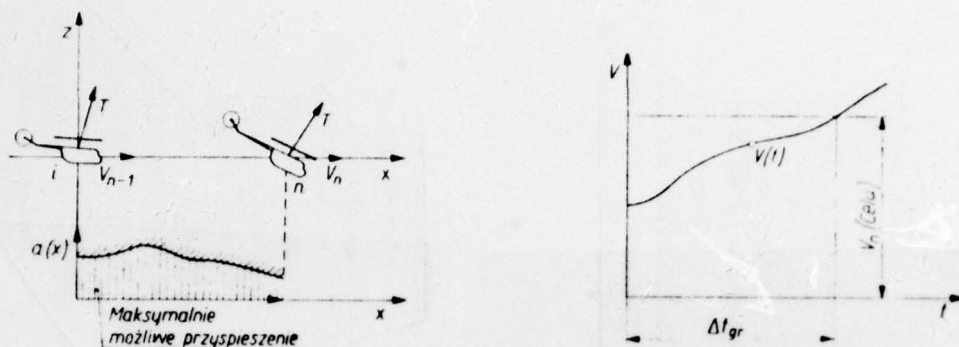for correction of the control steering system); 3 - the limiting

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Figure 16.   Evaluation of Δt_{gr} using as an example the horizontal flight acceleration.
Key: Maximum possible acceleration.

possibility of a change in motion in the immediate future(for estimating of the necessary time for prediction of Δt_{gr}).

2.3.   Distribution of Excess Available Engine Power

Program PR_n often involves the problem of how to allot engine power in order to obtain the conditions given at the end of a segment for the maneuver under consideration.   In the general case the distribution of power in three-dimensional motion can be in 5 directions: 1) for climbing; 2) for horizontal acceleration; 3) for lateral acceleration; 4) for accelerating the rotor; 5) for the tail propeller in case of rotation around the vertical axis.   The necessary power changes cause changes in the flow around the rotor and in the rotor loads during an instantaneous state of helicopter equilibrium during a maneuver.   The kinetic energy of the rotor inertial system can be considered as a part of the available power because of the specific nature of its source.

The shares of the excess power a_u in each of the 5 above-mentioned directions can be presented as projections of a unit vector of control in a five-dimensional system.
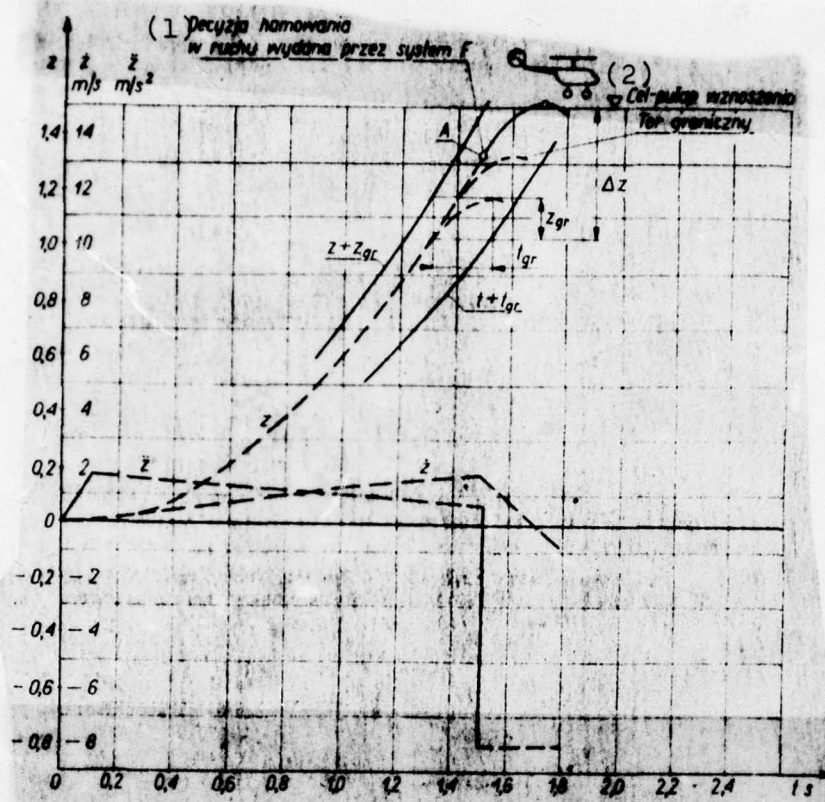
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Figure 17. Function of system F in single-parameter helicopter motion.
Key: (1) Decision to brake in motion taken according to system F; (2) Goal - climbing ceiling, Limiting path.

For the first approximation one can often apply an approximative analytical formula (e.g. in [7], [10]) where, assuming average values for the flight phase parameters and solving the system of equations for uniformly accelerated motion, one can compute approximative a_u values. They assure achievement of the parameters required in a definite time, allowing for their continuous correction in each i-step.

2.4. The Problem of Transient States

In order to carry out the flight task it is necessary in the PR_n program to connect individual maneuvers with transient phases. The condition is that the final parameters of the preceding segment become
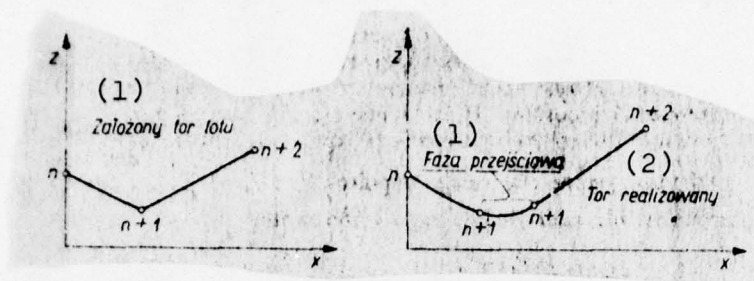
Figure 18.   Figure 19.

Figure 18. Assumed flight path.
Key: (1) assumed flight path.

Figure 19. Isolated transitional
phase.
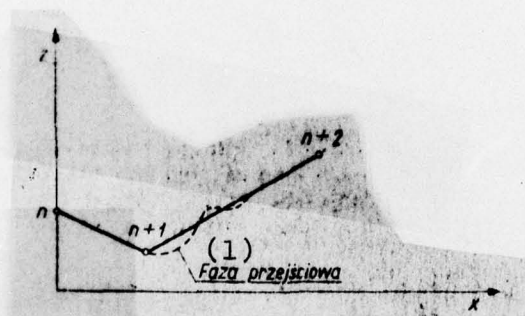Key: (1) Transitional phase; (2)
realized path.

Figure 20. The transitional phase
included in the beginning of the
next segment of the maneuver.
Key: (1) Transitional phase.

the initial conditions of the following segment. In this case there are
2 ways possible: 1) employment of an isolated transient phase during
which the above-mentioned condition of the conformability of the final
parameters of the preceeding segment with the initial parameters of
the following segment according to Figure 19 is realized, 2) inclusion
of a transient phase in the beginning of a subsequent segment (Figure
20).

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The second way is more often in use because of the greater efficiency of maneuvers (the maneuver becomes more compact). In the phase of connection of procedures, the beginning of the following segment should be assumed as a function of the goal, and the factor adjusting the helicopter flight parameters to the conditions of the following segment is the movements of the flight controls at a given control rate.

2.5. The Problem of Tolerance in Achieving the Final Parameters of a Segment

The final values of motion must be achieved with a certain tolerance. The tolerance value and direction should also be connected with the kind of maneuver assumed. The tolerance value (generally considerable) is assessed individually on the basis of experience, since excessively tight intervals of the tolerance value prolong the computation time (additional iterations) as well as the time for completing the maneuver.

2.6. The Problem of Control Impulse Selection

In the control program (when the rotor thrust impulse is being chosen) the impulses $\Delta\vartheta_i$ of the motion of the helicopter flight controls are evaluated ($\Delta\vartheta_i$ is symbolically understood as $\Delta\vartheta_0$, $\Delta\vartheta_y$, $\Delta\vartheta_x$, $\Delta\vartheta_r$).

The following basic constraints must not be exceeded during step i:

1) available power $P_r - P_n > 0$,

2) rotor lift (take-off) $T_{kr} - T > 0$,

3) design constraints of each channel of control $\vartheta_{min} < \vartheta < \vartheta_{max}$ (applies to collective pitch, cyclic pitch of inclination and tilt tail propeller pitch);

4) the requirements of the flight comfort, e.g. $\varphi > -15°$ (slope of passenger helicopter floor) or $|a| < 0.5 g$

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(the vertical acceleration);

5) $d\vartheta_i/dt < (d\vartheta_i/dt)_{dop}$.   of the permissible rate of control.

There can be other constraints dictated by the specifics of the maneuver performed. From procedure D one can obtain, for a given rotor load level and flight state computed from the equilibrium conditions (the RS procedure), steering derivatives (linearized for a low impulse) of the type $d_P = \dfrac{dP}{d\vartheta_i}$, $d_T = \dfrac{dT}{d\vartheta_i}$, etc.

Hence, the maximum control margins can be evaluated:

$$\Delta\vartheta_{i(P)} = \frac{P_r - P_n}{d_P} \; ; \quad \Delta\vartheta_{i(T)} = \frac{T_{kr} - T}{d_T}$$

One choses the minimum margin $\Delta\vartheta_{imin}$, which insures that the given constraints in step i are not exceeded. On obtaining $\Delta\vartheta_{imin}$, one can evaluate the value of the increase in the thrust component:

$$\Delta T_i = d_T \, \Delta\vartheta_{imin}$$

This is a first approximation, since the positions of the wobble plate are precisely determined at the beginning of step i and the end of i+1. Every step thus begins with the values $T_{x, y, z}$ and $\vartheta_{0x,y}$ of determined through solution the inverse problem. It provides a complete picture of the pattern of forces and motions with the flight controls as a function of time during the solution of the flight problem. In practice it is sufficient to consider the coupling of $T_x(\vartheta_0, \vartheta_y)$ and $T_z(\vartheta_0, \vartheta_y)$, while the coupling with ϑ_x and the dependence on $T_y(\vartheta_0, \vartheta_x)$ can be omitted.

From the physical point of view this means that the rotor inclination (in order to *increase* component T_x) requires the correction (increase) of the collective pitch, or, when the collective pitch (in order to increase force T_z) is increased, a correction of the rotor inclination (decrease in the plate inclination) is required during oblique

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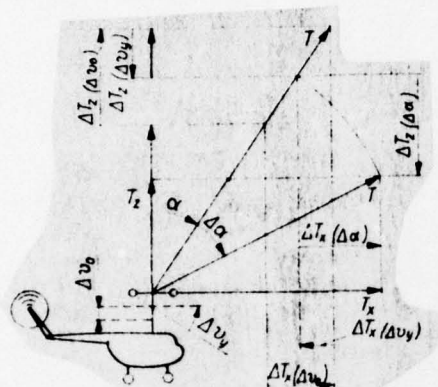flight in order to maintain equilibrium conditions along the x-axis (Figure 21).

Figure 21. Diagram of load impulses by the rotor, depending on the action of the control system plane x, z.

For given values of ΔT_z and ΔT_x the evaluation of increments $\Delta \vartheta_0$ and $\Delta \vartheta_y$ can be obtained through the solution of system of 2 equations with 2 unknowns:

$$\Delta T_z = \frac{dT_z}{d\vartheta_0}\Delta\vartheta_0 + \frac{dT_z}{d\vartheta_y}\Delta\vartheta_y + T_z(\Delta\alpha)$$

$$\Delta T_x = \frac{dT_x}{d\vartheta_0}\Delta\vartheta_0 + \frac{dT_x}{d\vartheta_y}\Delta\vartheta_y + T_x(\Delta\alpha)$$

where the increase in the thrust vector inclination is

$$\Delta\alpha = f(\Delta\vartheta_y, \Delta\vartheta_0)$$

The control system action brings about corresponding increases in the thrust components in relation to the system connected with the helicopter. Therefore, in order to analyze the flight path in the system connected with the earth, it is necessary to consider the change in the reference system.

3. Exemplary Illustration

3.1. A study of the Operation of Elements of the Proposed Method to Analyze Non-Stationary Helicopter Motion Taking as an Example a Vertical Maneuver [9]

The assigned physical behavior of the phenomenon is as follows: the helicopter climbs vertically from point Z_1 to point Z_2,

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descends from point Z_2 to point Z_3, and climbs to point Z_4. The points designated as $Z_n(n=1, 2, 3, 4)$ are characteristic points of the flight path (Figure 22).
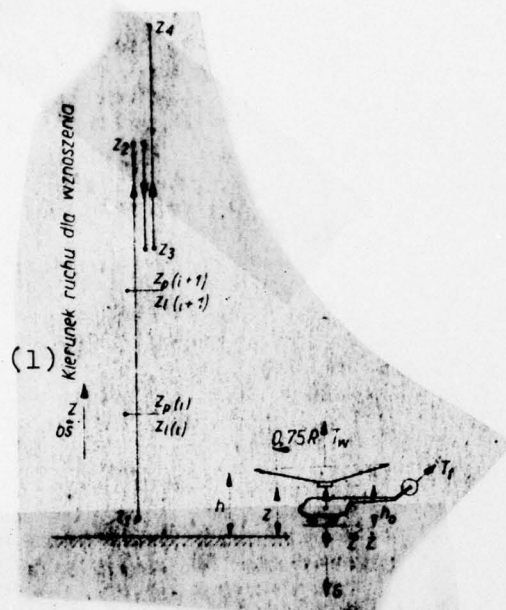

Figure 22. Diagram of symbols for single-parameter control motion of helicopter.
Key: Direction of motion for climbing.

The vertical path is divided into segments with equal time intervals Δt. The motion is solved on a "step-by-step" basis. The beginning of a step is designated z_i and its end z_{i+1}. Due to the discontinuity of the descreet system, at the beginning and end of a step one can distinguish the left side of section z_l and the right side of section z_p which are directed along the direction of motion. The solution is based on a group of typical procedures combined in a common system. In one computation step the right side of step $i-z_{p(i)}$ and the transition to $z_{l(i+1)}$ are computed. The principal block connecting the entire system is the PR_n program, in which there is a coded way of realizing helicopter motion in the flight segment under consideration. Here one choses a suitable increase in the collective pitch of the wobble plate designated as $\Delta\vartheta_0$. The program employs auxiliary procedures and systems, such as: procedure Pn,ϑ (computation of the necessary power and of the wobble plate position); procedure KR (computation of the parameter changes during one step); system F (the area penetration in the immediate future and possibly the correction of the assumed principle of control); procedure D (control gradients), etc. The solution of the helicopter motion is shown on Figure 23.
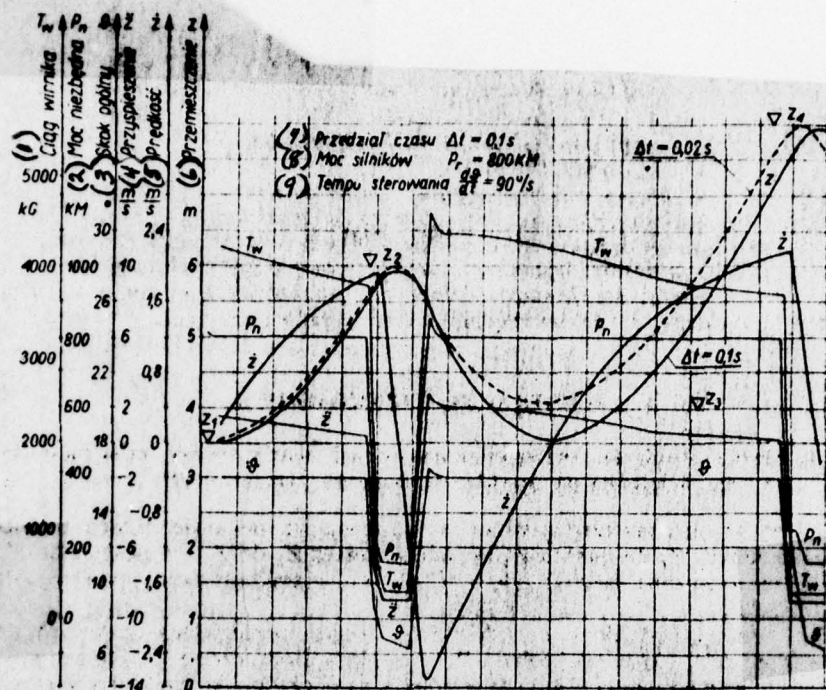
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Figure 23. Pattern of changes in parameters of
motion, control function and loading during a
simulated single-parameter helicopter maneuver
in the perpendicular.
Key: (1) Rotor thrust; (2) necessary power;
(3) total pitch; (4) acceleration; (5) speed; (6)
displacement; (7) time partition; (8) engine power;
(9) steering rate.

3.2. Simulation of the Helicopter
Interrupted Take-Off.

The method usefulness was verified by modelling a complex heli-
copter problem – a take-off with engine failure [10].

The simulation model of controlled flight, as applied to the
analysis of the helicopter take-off, includes certain simplifications,
namely:

- only two-dimensional motion in the plane symmetry x, z is considered

- the helicopter is considered as a material point (the equilibrium of moments in relation to the transverse y axis is ignored);

- it is assumed that the helicopter pitch is approximately equal to the thrust vector;

- in view of the low analyzed flight speeds, the computational algorithms omit specifics which are of importance for high flight speeds;

however, the following must be considered:

- the ground effect as a function of the flight speed and the influence of the inclination of the rotor plane in relation to the ground;

- optional structural, pilotage, aerodynamic, flight-comfort and other constraints which are essential from the point of view of helicopter take-off technique;

- the tolerance for execution of maneuvers in the take-off phase which correspond to the pilot's level of perception in estimating actual flight parameters.

In the computational program, a special control program has been worked out for every typical flight segment; it evaluates control impulses, allows for constraints, obeys principles of flight on the given segment and monitors achievement of the goal. This program uses standard procedures, typical for all flight segments, such as: the evaluation of the helicopter momentary equilibrium in unsteady flight; the control system position in relation to the helicopter.

The result of the completed simulation process is the profile of the helicopter motion parameters which is shown on Figure 24 where:
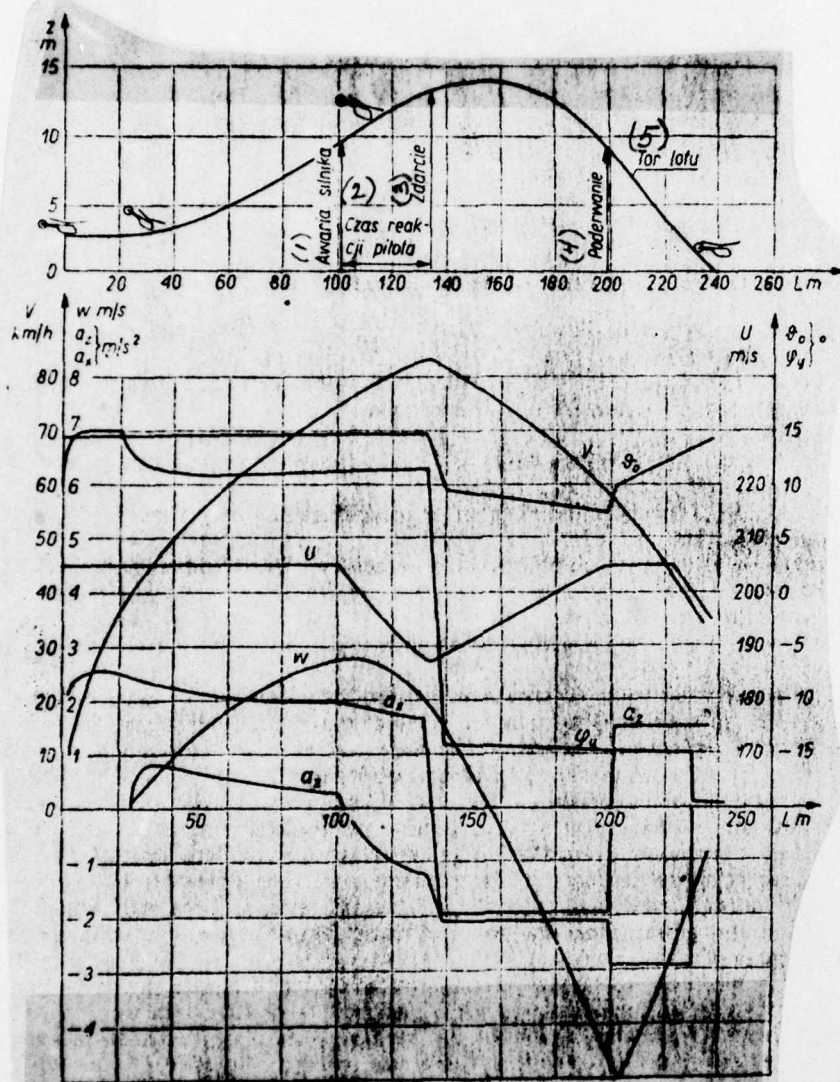
Figure 24. Interrupted take-off-landing G = 4450 kG, H=0, AW; profile of changes in parameters of helicopter motion.
Key: (1) Engine failure; (2) time for pilot reaction; (3) pulling up; (4) pulling off; (5) flight path.

V is the helicopter horizontal speed;

w is the vertical speed;

a_x and a_z are the horizontal and vertical accelerations of the helicopter;

ϑ_0 is the collective pitch angle;

ϕ_y is the helicopter pitch;

U is the blade tip speed.

4. Suggestions

The above mentioned employment of the similar analyses can be also applied to other helicopter flight problems. The information obtained in this way gives a comprehensive picture of the pattern of effects during the task being performed, with easy computation of different variants of the flight.

Errors in the motion evaluation arising from the adoption of an approximate helicopter model are of no special significance, since in reality the repetition of maneuvers, because of different flight conditions, pilotage techniques, etc., is not a strict rule. The spread is generally large - larger than the error of the method.

In view of the ever-increasing popularity and utilization of the method of representing reality through the fullest possible simulation, in order to solve the problems of the flight simulation (helicopter, glider, airplane) it is necessary to undertake and direct studies to determine the perception of the flight parameters in the "man-machine" system.

The data, mostly in the field of pilot psychophysiology (e.g. reaction time, visual estimation of flight altitude, etc) must be

specified numerically, since they represent the input data in the computational program.

BIBLIOGRAPHY

1. *Berman H., Gran R.*: An organized approach to the digital autopilot design problem. AIAA Paper 1973, No. 848, 1—11 (Aviastrojenije № 8, 27.II.1974).
2. *Hoffman W. C., Zvara I., Bryson A. E.*: A landing approach guidance scheme for unpowered lifting vehicles. Journal of Space Aircraft and Rockets, Vol. 7, No. 2, February 1970.
3. *Hoffman W. C., Zvara I., Bryson A. E., Ham N. D.*: Automatic guidance concept for VTOL aircraft. Journal of Aircraft Vol. 8, No. 8, August 1971.
4. *Kriechbaum G. K. L., Stineman R. W.*: Design of desirable airplane handling qualities via optimal control. Journal of Aircraft 1972, Vol. 9, No. 5, p. 365—369 (Aviastrojenije № 5, 7.II.1973).
5. *Connor G. L., Trautwein W., Livingston I. M.*: Computerized optimal control system design form reusable and expendable boost vehicles. AIAA Paper 1972, No. 98, p. 1—13 (Astronawtika i rakietodinamika No. 23, 21.VI.1972).
6. *Shapley I. I., Kyker R. A., Kerrel K. R.*: The development of an improved methods of conducting height-velocity testing on rotary wing aircraft. Journal American Helicopter Society 1970, Vol. 15, № 2, p. 30—38.
7. *Szumański K.*: Start śmigłowca z wykorzystaniem energii kinetycznej inercyjnego układu wirnika nośnego. Prace Instytutu Lotnictwa nr 43, 1970 r.
8. *Szumański K.*: Obliczenia aerodynamiczne śmigłowca. Wyd. wewn. nr arch. PP-28, 1973 r.
9. *Szumański K.*: Zagadnienia niestacjonarne mechaniki lotu śmigłowca. Wyd. wewn. nr arch. ST-7, 1974 r.
10. *Szumański K.*: Modelowanie toru przerwanego startu śmigłowca. Wyd. wewn. nr arch. PP-142, 1976 r.

Summary

A method for digital simulation of helicopter flight is submitted, thus enabling representation of the way in which manoeuvres are performed.

An attempt is made to construct a model of the pilot's actions including the general survey of the situation, the forecast of the motion of the helicopter, the decision and the assessment of the flight parameters.

The digital simulation technique is used as ensuring greater freedom of choice of the computation model.

By dividing the period of the manoeuvre into small time intervals, the solution of the equation of motion of the helicopter is corrected for each successive step, so that a set of data describing the manoeuvre (usually for the end of the flight trajectory) is obtained without affecting the manoeuvre constraints.

As the computation technique is improved the usefulness of complete simulation of real physical processes becomes greater, thus enabling more detailed analysis, reduction of expensive experimental research and, in the case of aircraft applications, analysis of the limiting cases, experimental investigation of which is dangerous.

Such a simulation is particularly justified for helicopters, the description of the flight of a helicopter requiring many data, but may also be of use for the dynamic behaviour of other controlled objects (such as airplanes, gliders and motor-cars).

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